PERTH MODERN SCHOOL
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Test One

Semester One 2016
Year 12 Mathematics Methods
Calculator Free

Teacher:

_____ **Mr Staffe**
_____ **Mrs. Carter**
_____ **Mr Bertram**
_____ **Mr Roohi**
_____ **Ms Cheng**

Name:

- ***Complete all questions***
- ***Show all necessary working***
- ***Total Marks = 25***
- ***25 minutes***

1. [12 marks]

Find $\frac{dy}{dx}$ in each of the following, by using the appropriate rule.

(a) $y = (3x^2 - x)(x^3 - 4x^2 - 5x + 3)$ (Do not simplify) [2]

$\frac{dy}{dx} = (x^3 - 4x^2 - 5x + 3)(6x - 1) + (3x^2 - x)(3x^2 - 8x - 5)$ ✓ ✓

(b) $y = 2x - \sqrt{x} + 3\pi^3 + \frac{4}{x^2}$ (Leave with positive indices.) [2]

$\frac{dy}{dx} = 2 - \frac{1}{2}x^{-1/2} - 8x^{-3}$

$= 2 - \frac{1}{2\sqrt{x}} - \frac{8}{x^3}$ ✓✓

(c) $y = \frac{2x^3}{(5 - 3x^4)^2}$ (Do not simplify) [3]

$\frac{dy}{dx} = \frac{(5 - 3x^4)(6x^2) - 2x^3 \cdot 2(5 - 3x^4)(-12x^3)}{(5 - 3x^4)^4}$ ✓ ✓ ✓

(d) $y = \sqrt{x^4 - 3x^3 + 2}$ [3]

$\frac{dy}{dx} = \frac{1}{2}(x^4 - 3x^3 + 2)^{-1/2} \cdot (4x^3 - 9x^2)$ ✓ ✓

$= \frac{4x^3 - 9x^2}{2\sqrt{x^4 - 3x^3 + 2}}$ ✓

(e) $y = \sqrt{u^2 - 3}$ using the chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, where $u = 2x^3 + 3$ [2]

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{2}(u^2 - 3)^{-1/2} \cdot 2u \times 6x^2$ ✓

$= \frac{2(2x^3 + 3) \cdot 6x^2}{2\sqrt{(2x^3 + 3)^2 - 3}}$ ✓

2. **[3 marks]**

Consider the function $f(x) = x^3 - 5x^2 - 8x + p$ where p is a constant.

(a) Determine where the local (relative) extrema points occur. [2]

$f'(x) = 3x^2 - 10x - 8$ ✓

$3x^2 - 10x - 8 = 0$

$(3x+2)(x-4) = 0$

$x = -\frac{2}{3}, 4$ ✓

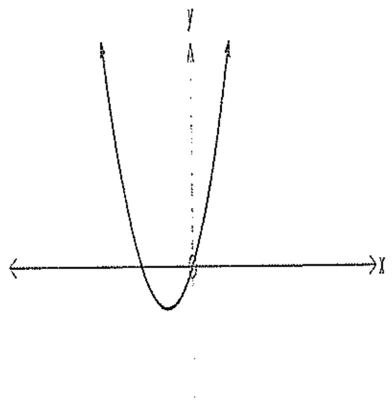
(b) What can we say about value of p given that two of the three roots are negative [1]

p is negative.

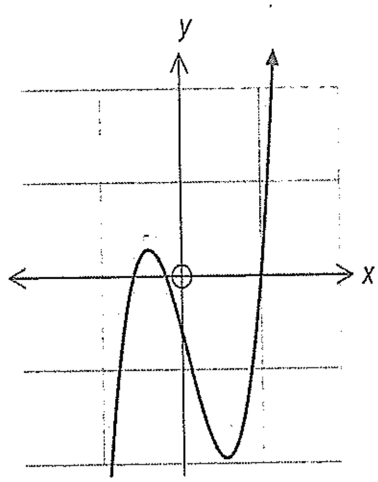
3. **[4 marks]**

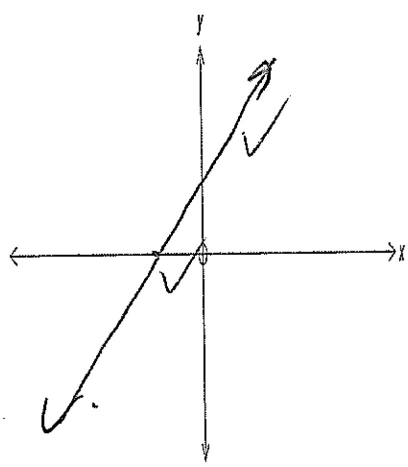
Draw a sketch below of each of the gradient functions formed by each of the following functions

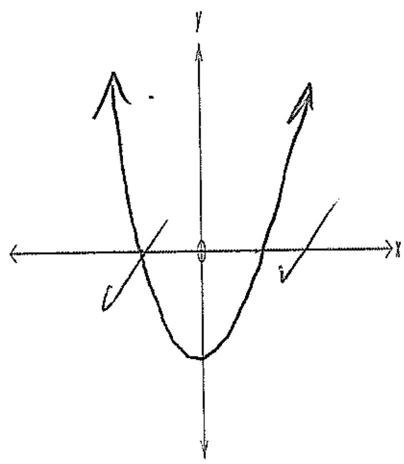
(a)



(b)

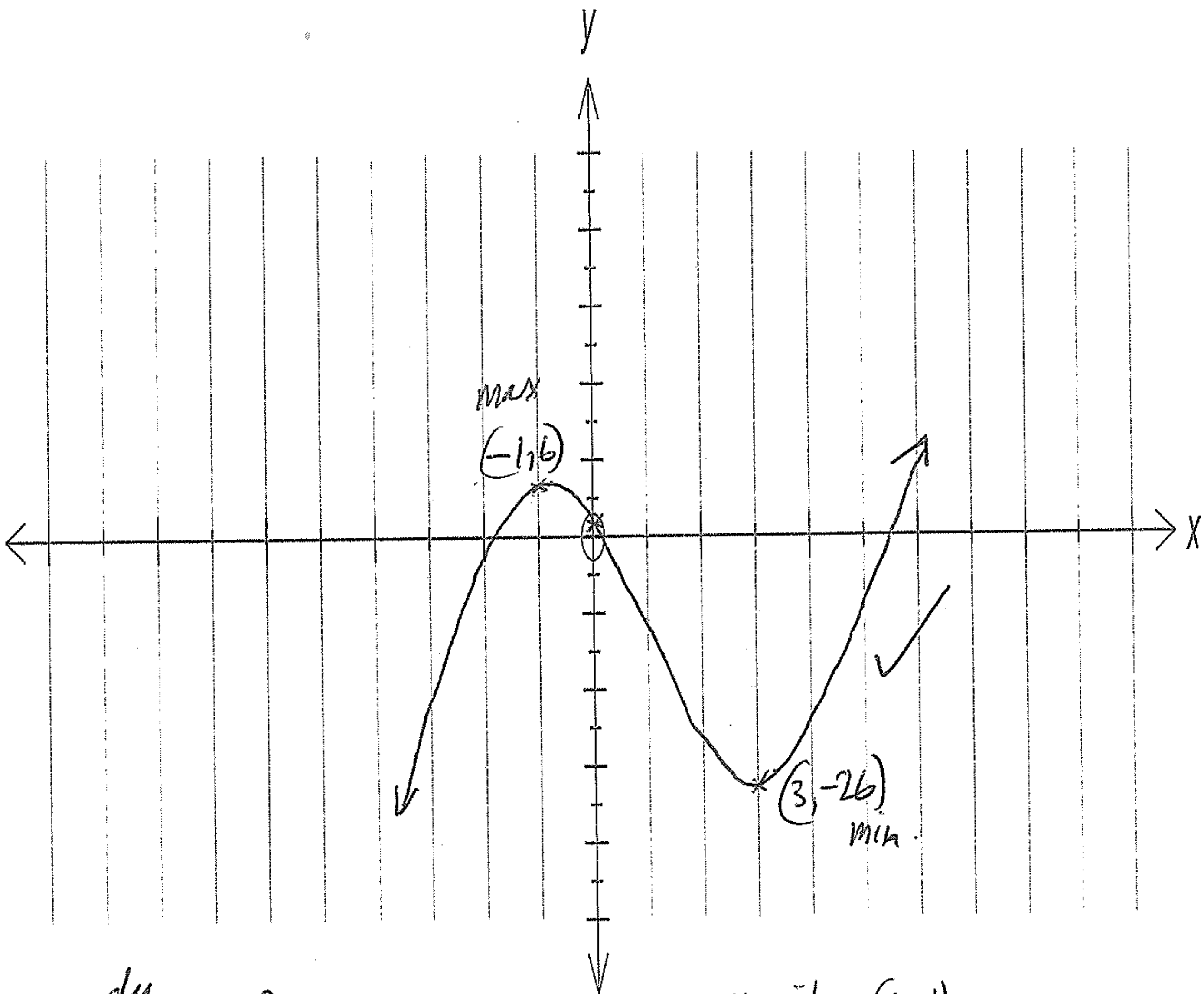






4. [6 marks]

Find the turning points, points of inflection and intercepts for the function $y = x^3 - 3x^2 - 9x + 1$. Then graph a sketch of the function on the axes provided below, clearly showing these key points.



$\frac{dy}{dx} = 3x^2 - 6x - 9$. ✓ y·int = (0, 1)

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$ ✓

$(x-3)(x+1) = 0$

$x = -1, 3. \rightarrow y = (-26), 6.$ ✓

$\frac{d^2y}{dx^2} = 6x - 6$. ∴ pt of inflection $x = (1, -10)$. ✓

When $x = -1$ $\frac{d^2y}{dx} < 0$. max.

When $x = 3$. $\frac{d^2y}{dx} > 0$ min ✓

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1. **[5 marks]**

A particle's position along the x-axis, in meters, is given by the function $s = 3t^3 - 5t + 9$.

(a) Find the Velocity and Acceleration of this particle when t = 2 seconds [3]

$v = 9t^2 - 5$ ✓

$a = 18t$ ✓

At $t = 2$ $v = 31\ m/s$, $a = 36\ m/s^2$ ✓

(b) When does the particle stop moving, and how far from the origin is it at this time? [2]

$9t^2 - 5 = 0$

$t = \sqrt{5}/3$ ignore -ve value. ✓

$s(\sqrt{5}/3) = 6.51\ m$.

Stops after $\sqrt{5}/3$ s at 6.51 m. ✓

2. [8 marks]

The volume of a certain rectangular box is given by the equation $f(x) = x^3 - 5x^2 - 8x + 48$.

(a) If the height of the box is $(4 - x)$ units, determine an algebraic expression for the area of the base of the box. [3]

$$\text{Area of base} = \frac{x^3 - 5x^2 - 8x + 48}{4 - x} \quad ✓✓$$

$$= -x^2 + x + 12 \quad ✓$$

(b) Calculate the value of x for which the volume is a maximum. [5]

$$f'(x) = 3x^2 - 10x - 8 \quad ✓$$

$$= (3x+2)(x-4) = 0$$

$$x = -\tfrac{2}{3},\ 4 \quad ✓$$

$$f''(x) = 6x - 10$$

$f''(-\tfrac{2}{3}) \leq 0$ max ✓

$f''(4) \geq 0$ min ✓

$\therefore$ max when $x = -\tfrac{2}{3}$ ✓

3. [7 marks]

(a) If the volume of a cylinder is given by $V = 2\pi r^3$, find the appropriate percentage change in V when r changes by $\frac{1}{2}$% [3]

$V = 2\pi r^3$

$\frac{dV}{dr} = 6\pi r^2 \quad \frac{\delta r}{r} - 0.005$

$\delta V \approx \frac{dV}{dr} \times \delta r$

$\frac{\delta V}{V} = \frac{dV}{dr} \times \frac{\delta r}{2\pi r^2}$

$= 3 \times 0.005$

$= 0.015 = 1.5\%$ change of 1.5%.

(b) If the volume of the solid generated by rotating a shaded region is given by $V = \pi[0.05h^5 + \frac{2}{3}h^3 + 4h]$, use the incremental formula, $\delta V \approx \frac{dV}{dh}\delta h$, to estimate the change in volume when h increases from 3 to 3.01. [4]

$\frac{dV}{dh} = \frac{\pi(h^4 + 8h^2 + 16)}{4}$ off classpad.

For small change on h $\frac{\delta V}{\delta h} \approx \frac{\pi(h^4 + 8h^2 + 16)}{4}$

$\delta V = \frac{\pi(3^4 + 8.3^2 + 16)}{4} \times (0.01)$

$= \frac{169\pi}{400}$

≈ 1.33 units.

The increase would be 1.33 units as h increase 3 to 3.01.

4. [5 marks]

Sketch the graph of y = f(x) given the data below:

(i) f(2) = - 9, f(- 4) = 27, f(-1) = 9

(ii) f′(2) = 0 and f″(2) > 0 min t.p at $x=2$

(iii) f′(−4) = 0 and f″(−4) < 0 max at $x=-4$.

(iv) f″(−1) = 0 Inflection when $x=-1$.

(v) f′(x) > 0 for x > 2, x < - 4

(vi) f′(x) < 0 for - 4 < x < 2

(vii) f (0) = 3